2. Mathematikschulaufgabe

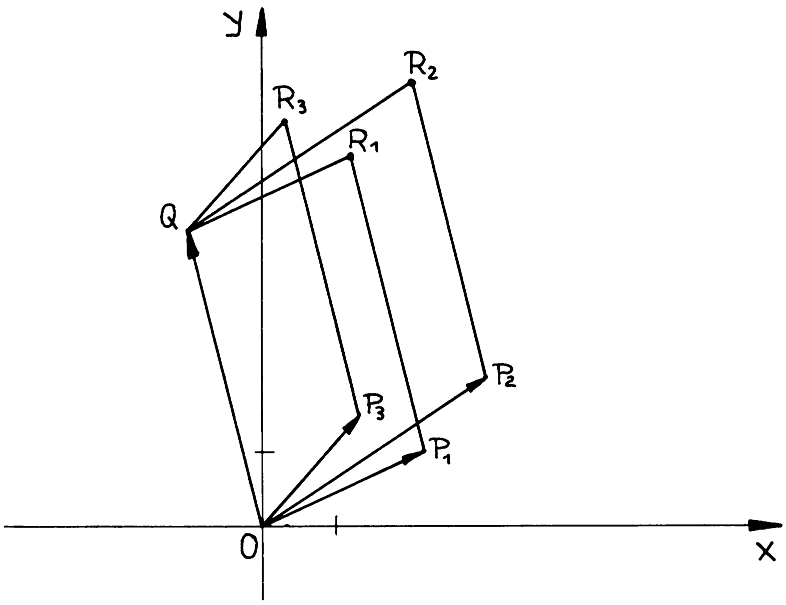
Klasse 10 / I

- Lösungen -

1. geg.: $\overrightarrow{OP} = \begin{pmatrix} 2 + \sin\mu \\ 2 - \cos^2\mu \end{pmatrix}$; $\overrightarrow{OQ} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$

1.1

μ	$10°$	$90°$	$225°$
$\overrightarrow{OP}$	$\begin{pmatrix} 2{,}17 \\ 1{,}03 \end{pmatrix}$	$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$	$\begin{pmatrix} 1{,}29 \\ 1{,}5 \end{pmatrix}$



1.2

$$A(\mu) = \left| \overrightarrow{OP} \ \ \overrightarrow{OQ} \right| \text{ FE}$$

$$A(\mu) = \begin{vmatrix} 2 + \sin\mu & -1 \\ 2 - \cos^2\mu & 4 \end{vmatrix} \text{ FE}$$

$$A(\mu) = (8 + 4\sin\mu + 2 - \cos^2\mu) \text{ FE}$$

$$\underline{\underline{A(\mu) = (-\cos^2\mu + 4\sin\mu + 10) \text{ FE}}}$$

$$A(\mu) = (\sin^2\mu - 1 + 4\sin\mu + 10) \text{ FE}$$

$$\underline{\underline{A(\mu) = (\sin^2\mu + 4\sin\mu + 9) \text{ FE}}}$$

| $\sin^2\mu + \cos^2\mu = 1$
| $-\cos^2\mu = \sin^2\mu - 1$

- Lösungen -

1.3

$$14 = \sin^2\mu + 4\sin\mu + 9$$

$$\sin^2\mu + 4\sin\mu - 5 = 0 \qquad \text{Substitution:}$$

$$x^2 + 4x - 5 = 0 \qquad \sin\mu = x$$

$$x_{1/2} = \frac{-4 \pm \sqrt{16+20}}{2}$$

$$\underline{x_1 = 1} \;\vee\; \underline{x_2 = -5}$$

$$\sin\mu = 1$$

$$\underline{\mu = 90^\circ} \qquad \underline{\underline{\mathbb{L} = \{90^\circ\}}}$$

1.4

$$\begin{array}{|l} 2 + \sin\mu = x \\ \wedge\; 2 - \cos^2\mu = y \end{array}$$

$$\begin{array}{|l} \sin^2\mu + \cos^2\mu = 1 \\ -\cos^2\mu = \sin^2\mu - 1 \end{array}$$

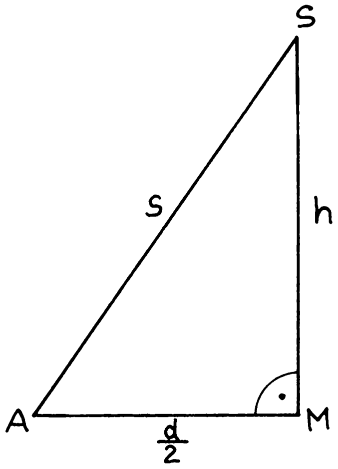
$$\begin{array}{|l} \sin\mu = x - 2 \\ \wedge\; 2 + \sin^2\mu = y \end{array}$$

$$2 + (x-2)^2 = y$$

$$\underline{\underline{y = x^2 - 4x + 5}}$$

- Lösungen -

2.1 geg.: $h = \sqrt{46}$ cm ; $s = 8$ cm



halbe Diagonalenlänge:

$\frac{d}{2} = \sqrt{s^2 - h^2} \quad | \cdot 2$

$d = 2\sqrt{64 - 46}$

$\underline{d = 2\sqrt{18} \text{ cm}}$

$d = a\sqrt{2}$

$a = \frac{2\sqrt{18}}{\sqrt{2}}$

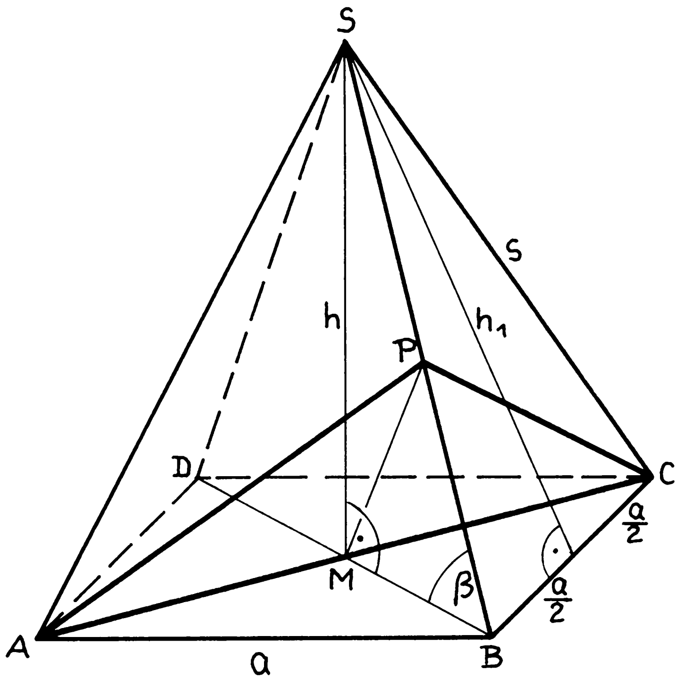
$\underline{\underline{a = 6\text{cm}}}$

$V = \frac{1}{3} \cdot a^2 \cdot h$

$V = \frac{1}{3} \cdot 36 \cdot \sqrt{46}$

$\underline{\underline{V = 81{,}4 \text{ cm}^3}}$

2.2



- Lösungen -

2.3 $\triangle MBS$:

$$\sin\beta = \frac{h}{s}$$

$$\sin\beta = \frac{\sqrt{46}}{8}$$

$$\underline{\underline{\beta = 57{,}97°}}$$

2.4

$$\cos\sphericalangle CBS = \frac{\frac{a}{2}}{s}$$

$$\cos\sphericalangle CBS = \frac{3}{8}$$

$$\underline{\underline{\sphericalangle CBS = 67{,}98°}}$$

$$\underline{\underline{\sphericalangle BCS = 67{,}98°}}$$

$$\underline{\underline{\sphericalangle BSC = 44{,}04°}}$$

2.5

$$A_M = 4 \cdot A_{\triangle BCS}$$

$$A_M = 4 \cdot \frac{1}{2} \cdot a \cdot h_1$$

$$A_M = 2 \cdot 6 \cdot 7{,}42 \text{ cm}^2$$

$$\underline{\underline{A_M = 89 \text{ cm}^2}}$$

Höhe im $\triangle BCS$:

$$h_1 = \sqrt{s^2 - \left(\frac{a}{2}\right)^2}$$

$$h_1 = \sqrt{64 - 9}$$

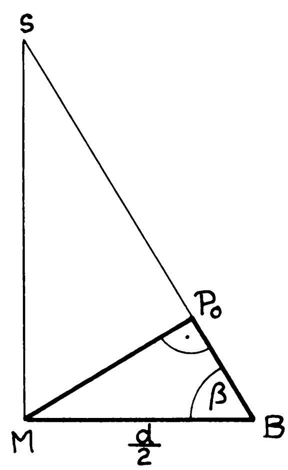
$$\underline{h_1 = 7{,}42 \text{ cm}}$$

2.6 Das $\triangle ACP_0$ hat minimalen Flächeninhalt, wenn die Höhe $\overline{MP_0}$ am kleinsten ist.
Dies ist der Fall für $\overline{MP_0} \perp \overline{BS}$

$$\sin\beta = \frac{\overline{MP_0}}{\frac{d}{2}}$$

$$\overline{MP_0} = \sqrt{18} \text{ cm} \cdot \sin 57{,}97°$$

$$\underline{\underline{\overline{MP_0} = 3{,}6 \text{ cm}}}$$



- Lösungen -

$$A_{\Delta ACP_0} = \frac{1}{2} \cdot \overline{AC} \cdot \overline{MP_0}$$

$$A_{\Delta ACP_0} = \frac{1}{2} \cdot 2\sqrt{18} \cdot 3{,}6 \quad cm^2$$

$$\underline{\underline{A_{\Delta ACP_0} = 15{,}26\ cm^2}}$$

$$A_{\Delta MBP_1} = \frac{1}{2} \cdot \overline{MB} \cdot \overline{BP_1} \cdot \sin\beta$$

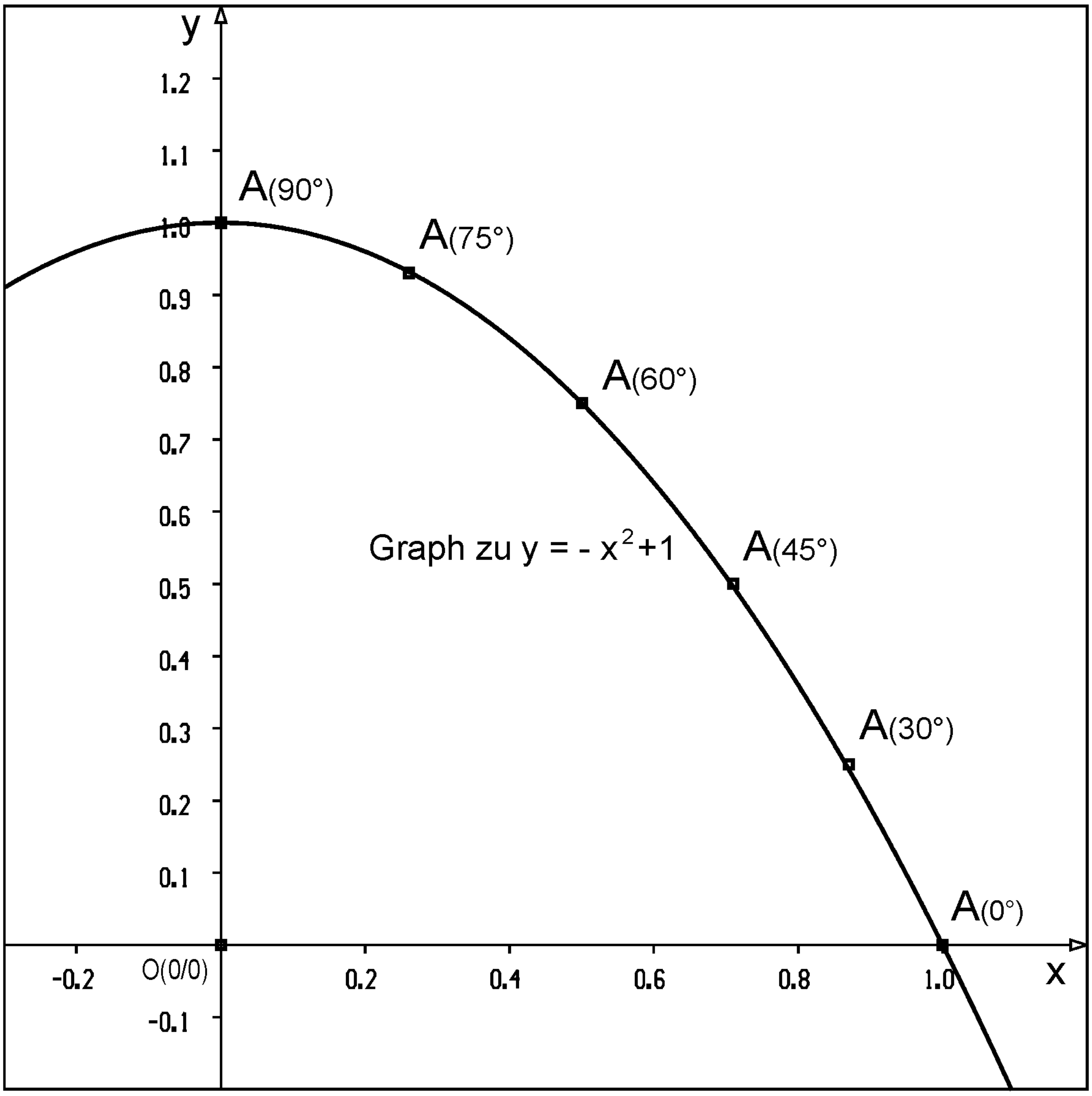
$$10\,cm^2 = \frac{1}{2} \cdot \sqrt{18}\ cm \cdot \overline{BP_1} \cdot \sin 57{,}97°$$

$$\overline{BP_1} = \frac{10\ cm^2}{0{,}5 \cdot 4{,}24 cm \cdot \sin 57{,}97°}$$

$$\underline{\underline{\overline{BP_1} = 5{,}56\ cm}}$$

- Lösungen -

3.1



$A_{(0°)}$ (1/0); $A_{(30°)}$ (0,87/0,25); $A_{(45°)}$ (0,71/0,5);

$A_{(60°)}$ (0,5/0,75); $A_{(75°)}$ (0,26/0,93); $A_{(90°)}$ (0/1)

3.2

$$x = \cos\alpha \quad \wedge \quad y = \sin^2\alpha$$

$$x = \cos\alpha \quad \wedge \quad y = 1 - \cos^2\alpha$$

$$y = 1 - x^2$$

$$\underline{\underline{\mathbf{y = - x^2 + 1}}}$$

